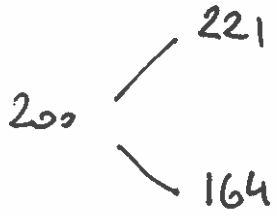
Problem Set 5

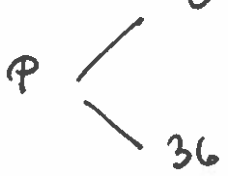
Problem 1



Stock

$$q = \frac{200\, e^{(0.08-0.04)\times 6/12} - 164}{221 - 164}$$

$$= 0.702$$

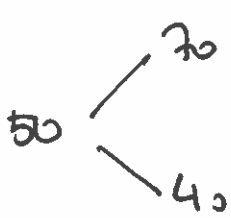


Put

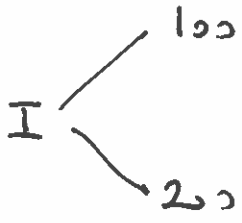
$$Put = (0\,q + 36\,(1-q))\, e^{-0.08\times 6/12}$$

$$= 10.29$$

Problem 2



Stock

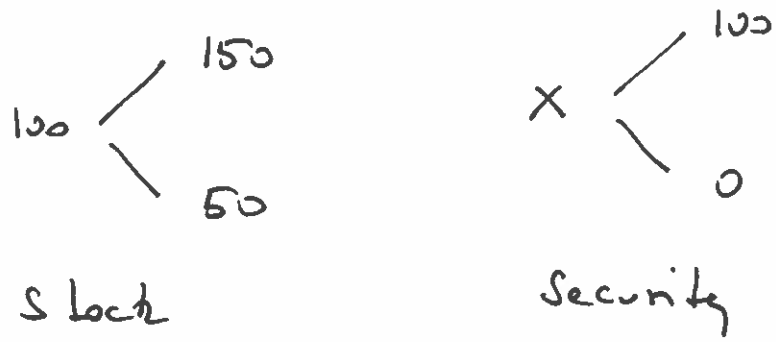


Instrument

$$Ns = \frac{100 - 200}{70 - 40} \times e^{-0.08\times 6/12} = -3.20$$

She should sell 3.20 shares.

Problem 3



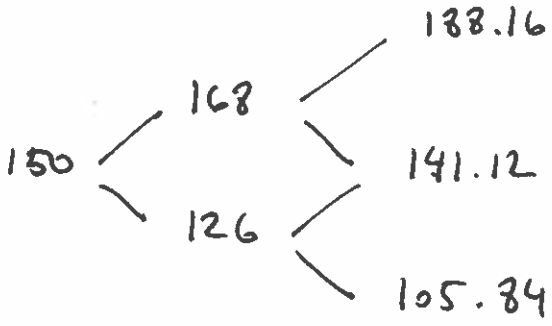
Stock Security

$$q = \frac{100\, e^{(0.05-0.05)\times 1} - 50}{150 - 50} = 0.5$$

$$X = (100\, q + 0\,(1-q))\, e^{-0.05} = 47.56$$

Problem 4

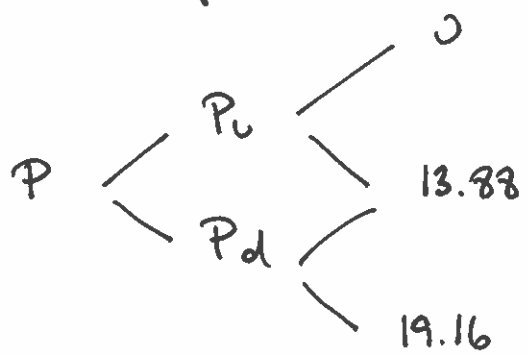
The binomial tree for the stock is



$$q = \frac{e^{0.10\times 3/12} - 0.84}{1.12 - 0.84}$$

$$= 0.6618$$

For the put



If $S = 168$ you cannot exercise.

$$\text{wait} = (0\,q + 13.88(1-q))\, e^{-0.10 \times 3/12} = 4.58$$

$$\Rightarrow P_0 = 4.58$$

If $S = 126$

$$\text{wait} = (13.88\,q + 19.16(1-q))\, e^{-0.10 \times 3/12} = 25.17$$

$$\text{exercise} = 155 - 126 = 29$$

$$\Rightarrow P_d = 29$$

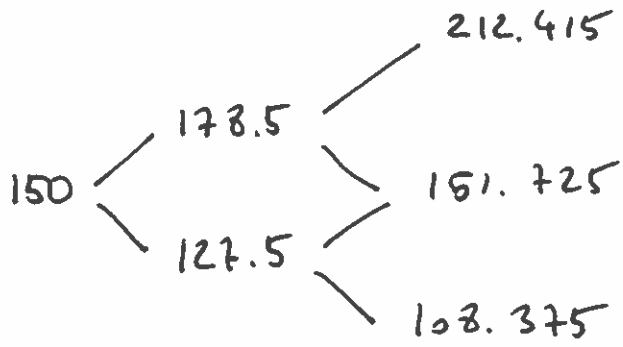
If $S = 150$

$$\text{wait} = (4.58\,q + 29(1-q))\, e^{-0.10 \times 3/12} = 12.52$$

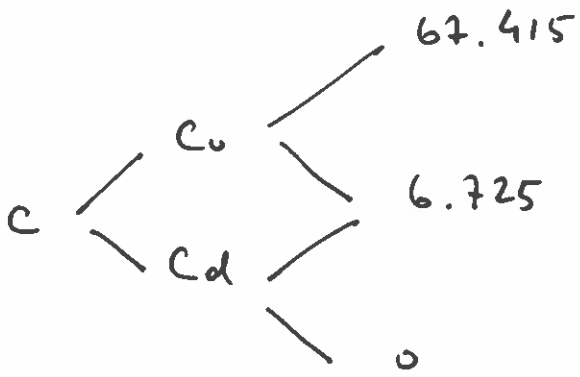
$$\text{exercise} = 155 - 150 = 5$$

$$\Rightarrow P = 12.52$$

Problem 5



$$q = \frac{e^{(0.08-0.06)\times 6/12} - 0.85}{1.19 - 0.85} = 0.4707$$



If $S = 178.5$

$$\text{Wait} = (67.415\,q + 6.725(1-q))\,e^{-0.08\times 6/12} = 33.91$$

$$\text{exercice} = 178.5 - 145 = 33.50$$

$\Rightarrow$ we wait so $C_u = 33.91$

If $S = 127.5$, we can only wait

$$C_d = (6.725\,q + 0(1-q))\,e^{-0.08\times 6/12} = 3.04$$

If $S = 150$

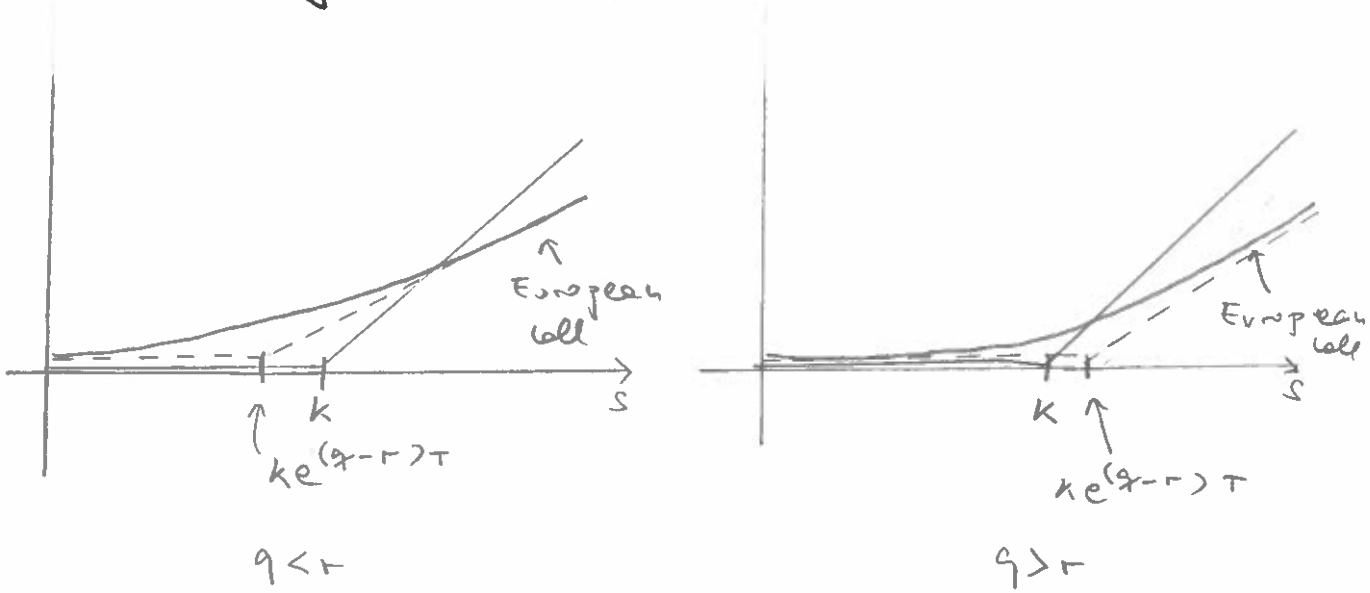
$$\text{wait} = (33.91q + 3.04(1-q))e^{-0.08 \times 6/12} = 16.88$$

$$\text{exercise} = 150 - 145 = 5$$

$$\Rightarrow \quad C = 16.88$$

Problem 6

False. If the dividend yield is positive, the time value of a European call will be negative for stock prices high enough regardless of the sign of the interest rate. This is because the slope coefficient of the lower bound is $e^{-qT} < 1$.



$q < r$ $\qquad$ $q > r$

Thus, it might be worthwhile to exercise early if the stock price is high enough.

Go Bears!

Problem 7

a. $P(Y > 50) = 1 - P_r(Y \leq 50)$

$= 1 - P_r\left(Z \leq \frac{\ln(50) - 4.30}{1.20}\right)$

$= 1 - P_r(Z \leq -0.323)$

$= 1 - 0.3745$

$= 0.6255$

b. $P_r(Y \leq 80) = P_r\left(Z \leq \frac{\ln 80 - 4.30}{1.20}\right)$

$= P_r(Z \leq 0.068)$

$= 0.5319$

c. $P_r(60 < Y \leq 70)$

$= P_r(Y \leq 70) - P_r(Y \leq 60)$

$= P_r\left(Z \leq \frac{\ln 70 - 4.30}{1.20}\right) - P_r\left(Z \leq \frac{\ln 60 - 4.30}{1.2}\right)$

$= P_r(Z \leq -0.043) - P_r(Z \leq -0.171)$

$= 0.4920 - 0.4129 = 0.0791$

↑ I used $\phi(-0.22)$ here.

Problem 8

$$E\, S_T = 229\, e^{0.108} = 225.12$$

Problem 9

$$E \ln S_T = \ln(102) + \left(0.15 - \frac{1}{2}\, 0.62^2\right) \times \frac{10}{12}$$

$$= 4.5898$$

$$\text{St. Dev.}(\ln S_T) = 0.62 \sqrt{10/12} = 0.5660$$

$$\Pr(S_T \leq 92) = \Pr\left(z \leq \frac{\ln 92 - 4.5898}{0.5660}\right)$$

$$= \Pr(z \leq -0.1202)$$

$$= 0.4522$$

Problem 10

$$E\ln(S_T) = \ln(193) + \left(0.18 - \frac{1}{2}0.49^2\right) \times 1 = 5.3226$$

$$\text{St. Dev}\left(\ln(S_T)\right) = 0.49\sqrt{1} = 0.49$$

$$Pr(S_T > 224) = 1 - Pr\left(z \leq \frac{\ln(224) - 5.3226}{0.49}\right)$$

$$= 1 - Pr(z \leq 0.1817)$$

$$= 1 - 0.5714$$

$$= 0.4286$$

Problem 11

$\frac{dS}{S}$ represents the instantaneous rate of return of the stock.

Problem 12

$$Y = S^6$$

$$Y_S = 6S^5$$

$$Y_{SS} = 30S^4$$

$$dY = 6S^5(\mu S\,dt + \sigma S\,dW) + \frac{1}{2}30S^4\sigma^2S^2\,dt$$

$$= 6\mu Y\,dt + 6\sigma Y\,dW + 15\sigma^2 Y\,dt$$

$$\frac{dY}{Y} = (6\mu + 15\sigma^2)\,dt + 6\sigma\,dW$$